A Geometrical Approach to Form in the Scherzo and Trio of Bartók's Fifth String Quartet

A simple metric implied by Tymoczko (2011, 65-66), which I call "voice-leading class" or "VLC," describes two voice note-against-note counterpoint using angles in the Cartesian plane. It works by representing pairs of notes (one from each voice) as ordered pairs. Vectors are formed by subtracting each pair of notes from the next pair, and the tails of these vectors are placed at the origin. The angles these vectors make with the positive x axis, measured counter-clockwise, give the VLC values (Figure 1). VLC values lie in the range (-180°, 180°].

Here I use the VLC technique to analyze the third movement of Bartók's Fifth String Quartet. I modify the technique by applying it to subsequent and/or not completely overlapping motives. Figure 2 illustrates the dependence of the categories of "virtual counterpoint" – similar, contrary, parallel, perfect contrary, and oblique – on VLC.

An interesting phenomenon occurs when two subsequent VLC values within one motive pair are symmetrical about the line of unit slope passing through the origin, and when the two motive forms within successive pairs alternate. In this case, the horizontal and vertical axes are exchanged, and pairs of VLC values alternate (Figures 3 and 4). This occurs in measures 9-13 and 19-22 (Figure 5) of the da Capo, providing contrast to the surrounding textures.

Furthermore, a histogram of VLC values (Figure 6) reveals not only generic structure – peaks at 45°, 135°, -135°, and -45°, corresponding to parallel or perfect contrary motion – but also reveals features characteristic of this piece. To see this, note that there are secondary peaks at VLC values of 53.13°, 36.87°, -126.87°, and -143.13°, corresponding to changes in the two motive forms of 4 and 3, 3 and 4, -4 and -3 , or -3 and -4.

These pairs of intervals characterize pitch-class sets of prime form (014); such sets are found at the end of the Scherzo (C♯2, A3, C4) and in measures 40-51 of the Trio (B♭2, C♯3, D3). According to Bayley (2001), Bartók posited an arch form for this quartet, with the first and last movements exhibiting B-flat centricities, and the third movement exhibiting C♯–E–C♯ centricities (there is a cadence in C♯ minor at the end of the third movement). Note that the second and fourth movements of the piece do not explicitly participate in the arch form. With the B♭ centricities in the first and final movements, and the paired C# centricities in the third movement, the (014) set in the Trio therefore completes a governing pc set, spelled out both locally (in the Trio) and globally (in the first, third, and final movements). This interpretation provides a possible way to augment Bartók's explanation of the third movement as being centered on C♯–E–C♯.

Thus, a modification of a simple geometrical metric based on work by Tymoczko, combined with pc set analysis, serves to delineate the form of the Scherzo and Trio of Bartók's Fifth Quartet, which serves, in turn, as a lynchpin of the entire piece's form.

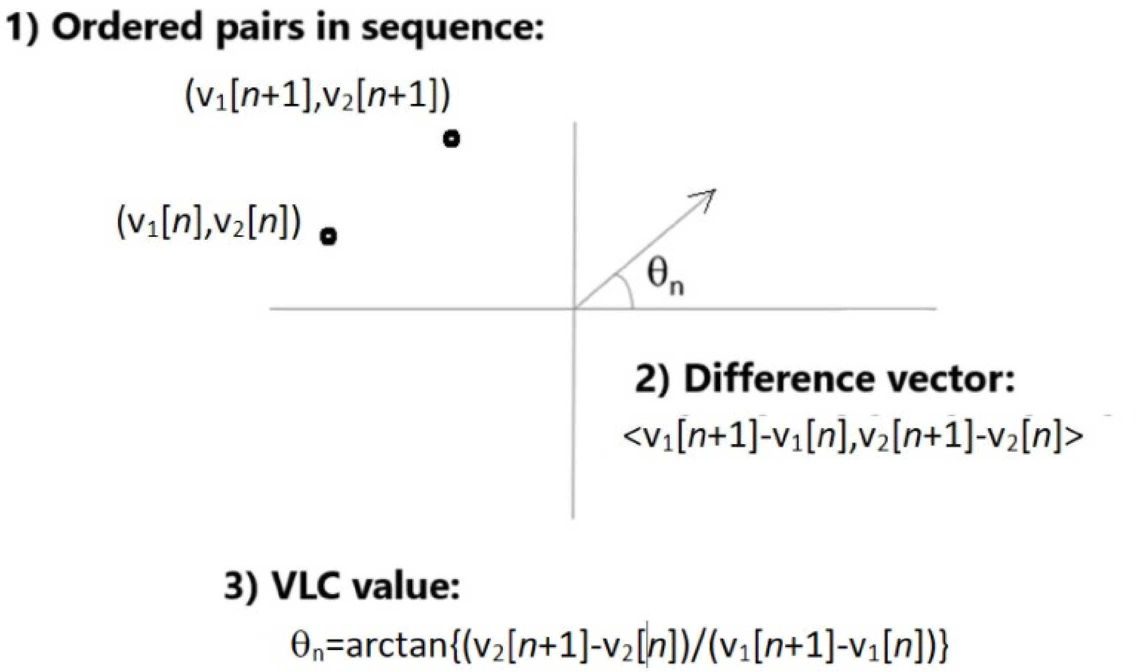

Figure 1. Construction of the VLC metric. The variables "v_1" and "v_2" stand for "voice 1" and "voice 2". The variables n and n+1 represent order within each voice. Note that a modified arctan function is used in order to keep track of which quadrant the vector lies in.

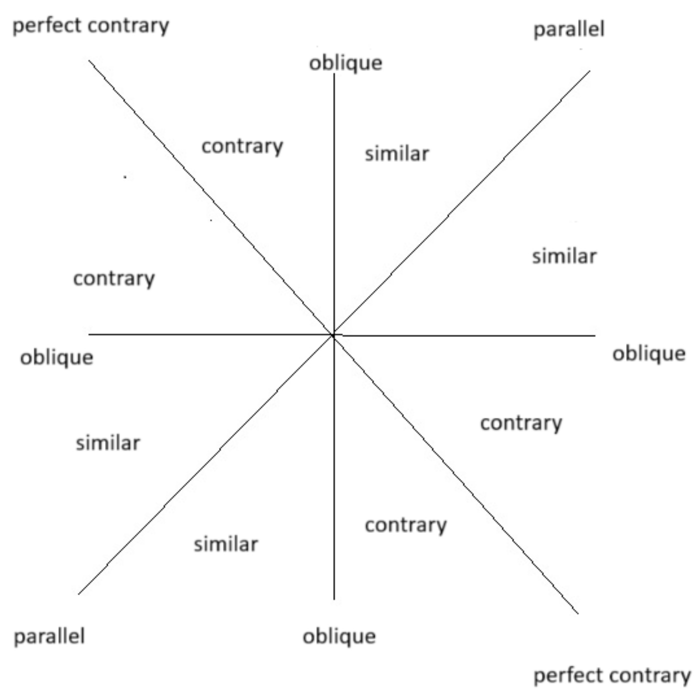

Figure 2. Relationships of types of virtual counterpoint to VLC. VLC values are measured positive in the counter-clockwise direction from the positive x direction, and lie in the range (-180°, 180°].

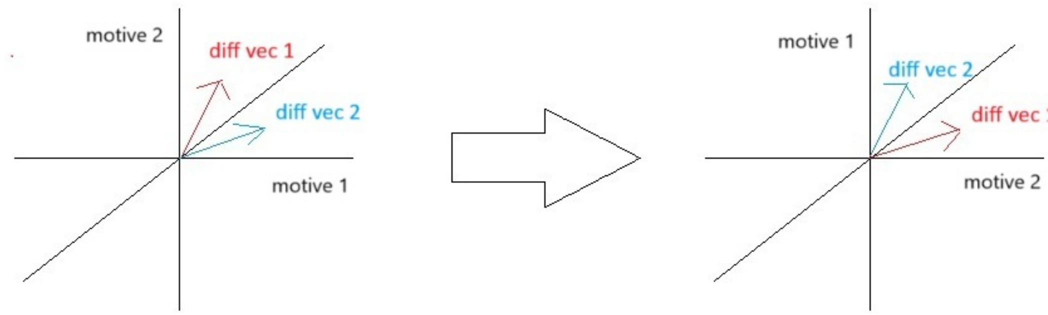

Figure 3. Symmetrically placed difference vectors (in color) within pairs of alternating motive forms (illustrated with large arrow). The alternation of motive forms exchanges axes and maps one difference vector onto the other.

45.00º	45.00º	45.00º
36.87º	51.13º	36.87º
51.13º	36.87º	53.13º
-126.87º	-143.13º	-126.87º
-143.13º	-126.87º	-143.13º
-135.00º	-135.00º	-135.00º
-135.00º	-135.00º	-135.00º

Figure 4. VLC values for alternating motive forms with pairs that are reflected about the line with unit slope that passes through the origin. These pairs alternate.

Figure 5. m. 17-28 of da Capo (alternating motives can be found in m. 19-22)

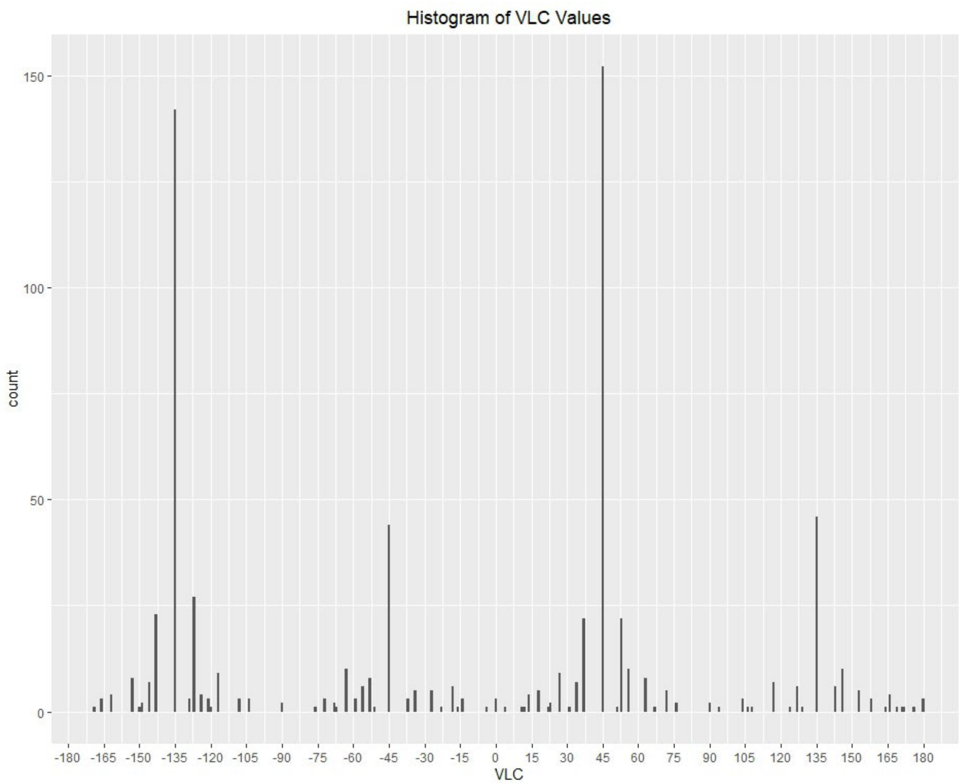

Figure 6. Histogram of VLC Values. Vertical axis is counts. Horizontal axis is VLC value measured in degrees.

References

Antokoletz, Elliott. *The music of Béla Bartók: a study of tonality and progression in twentieth-century music*. Univ. of California Press, 1984.

Bayley, Amanda. "The string quartets and works for chamber orchestra." In *The Cambridge companion to Bartók,* edited by Amanda Bayley, 151-174. Cambridge companions to music. Cambridge University Press, 2001

Bent, Ian. "Steps to Parnassus: contrapuntal theory in 1725 precursors and successors." In *The Cambridge history of Western music theory*, edited by Thomas Christensen, 554-602. Cambridge University Press, 2010

Fuller, Sarah. "Organum-discantus-contrapunctus in the Middle Ages." In *The Cambridge history of Western music theory*, edited by Thomas Christensen, 477-502. Cambridge University Press, 2010

Fux, Johann Joseph, *The study of counterpoint from Johann Joseph Fux's Gradus ad parnassum*, edited and translated by Alfred Mann. Norton, 1971

Nolan, Catherine. "Music theory and mathematics." In *The Cambridge history of Western music theory*, edited by Thomas Christensen, 272-304. Cambridge University Press, 2010

Schubert, Peter. *Modal counterpoint, Renaissance style*. Oxford University Press, 1999.

Stevens, Halsey. *The life and music of Béla Bartók*. Oxford University Press, 1993.

Tymoczko, Dmitri. *A geometry of music: Harmony and counterpoint in the extended common practice*. Oxford University Press, 2011.